2 JEE ADVANCED MOCK TESTS

SOLUTION OF PRACTICE TEST - 2

PAPER – 1

CHEMISTRY

Sol.1 [b]

Sol.2 [b, d]

$$\overset{1^{O}}{CH_3} - \overset{2^{o}}{CH} - \underset{3^{o}}{\overset{\overset{1^{O}}{CH_3}\,|}{CH}} - \overset{2^{o}}{CH_2} - \overset{1^{o}}{CH_3} \xrightarrow[hv]{Cl_2}$$

$$CH_3 - CH_2 - \underset{*}{\overset{CH_3\,|}{CH}} - CH_2 - CH_2 - Cl$$

$$2^{n} = 2^{1} = 2$$

$$+$$

$$CH_3 - CH_2 - \underset{*}{\overset{CH_3\,|}{CH}} - \underset{*}{\overset{Cl\,|}{CH}} - CH_2 - Cl$$

$$2^{n} = 2^{2} = 4$$

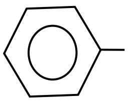
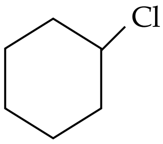
$$CH_3 - CH_2 - \underset{|\,Cl}{\overset{CH_3\,|}{C}} - CH_2 - CH_3$$

$$+$$

$$CH_3 - CH_2 - \overset{CH_2-Cl\,|}{CH} - CH_2 - CH_3$$

Total (8) Product

Sol.3 [c, d]

Because  $CH_2 - Cl$ and 

not have any chiral –*C* so that they gives same product by SN^1, SN^2

Sol.4 [a, b, c, d]
All contains symmetry, with chiral centre

Sol.5 [a, b, c, d]
Theory

Sol.6 [a, b, c, d]
After dilution $\left[H^+\right] = 10^{-2} \Rightarrow pH = 2$
Let *V* litre solution of $pH = 2$ is added in original solution so that pH remains fixed.

$$\therefore \left[H^+\right] = \frac{10^{-2} x + V \ x \ 10^{-2}}{10 + V} = 10^{-2}$$

This result is independent of volume taken.

Sol.7 [a, c, d]

$$x_A P_A^\circ + x_B P_B^\circ = 700 \qquad \text{....(i)}$$

$$x_A'' P_A^\circ + x_B'' P_B^\circ = 0.30\, P_A^\circ + 0.70\, P_B^\circ = 600 \qquad \text{....(ii)}$$

If moles of *A* & *B* initially are *x* & *y* then

$$x = 0.75 \times \frac{2}{3}(x + y) + 0.30 \times \frac{1}{3}(x + y)$$

& $\quad x_A = \dfrac{x}{x + y} \quad or \quad x_B = \dfrac{y}{x + y}$

solving gives

$$x_A = 0.6,\ x_B = 0.4,\ P_A^\circ = \frac{2500}{3} torr$$

& $P_B^\circ = 500\ torr$

Sol.8 [a, b]

$k_1 = k_2$

$\Rightarrow \dfrac{2}{3} rd$ A has reacted for $\left[A\right] = \left[C\right] = \left[D\right]$

$$\therefore k_1 + k_2 = \frac{1}{t}\ In\ \frac{[A]_0}{\frac{1}{3}[A]_0}$$

$$\Rightarrow t = \frac{1}{k_1 + k_2} \ell n3 = \frac{1}{2k_1} \ell n3 = \frac{1}{2k_2} \ell n3$$

Sol.9 [a, b, c]

MCl_x

In each mole of MCl_x there are *x* moles of Cl^-

$\Rightarrow \quad [Cl^-] = x \times 0.01$

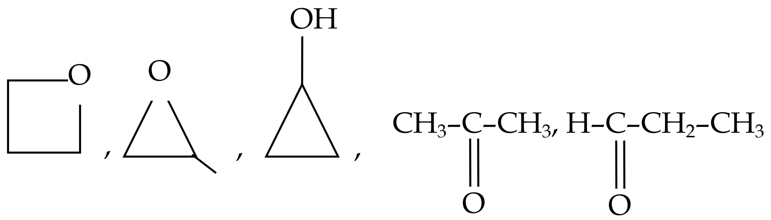
conc. of $[M^{x+}] = 0.01$

Sol.10 [b, c]

The pressure of NH_L will decrease due to addition of CO_2 (backward, shifting Le-chatelies's principle. The pressure of CO_2 will be more than $0.1\ atm$

Section - II [Q. 1 to Q. 10]

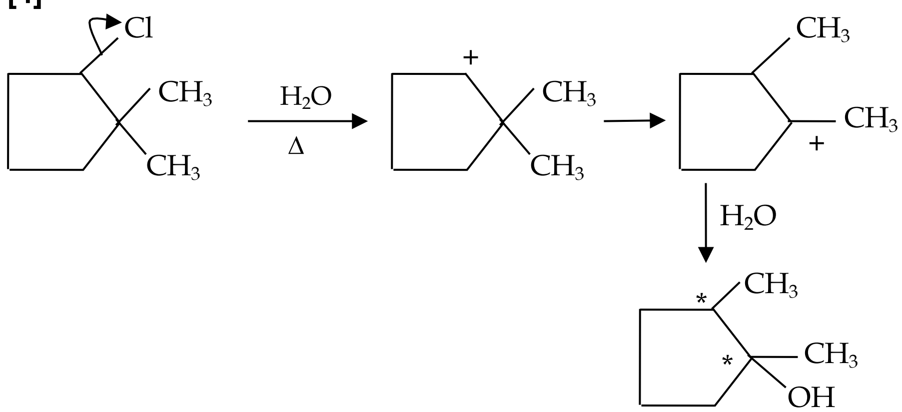
Sol.11 [9]



$CH_3-\underset{\displaystyle OH}{\underset{|}{C}}=CH_2, CH_3-CH=CH-OH,$

$CH_2=CH-O-CH_3$, $HO-CH_2-CH=CH_2$

Sol.12 [4]



$S.I. = 2^2 = 4$

Sol.13 [3]

Sol.14 [4]

(I, II, V, VI are correct)

Sol.15 [2]

$2Cl^- \rightarrow Cl_2 + 2e^-$

$Cl_2 + 2NaOH \rightarrow NaClO + NaCl + H_2O$

Moles of $NaClO$ required

$= \dfrac{10^6 \times 1 \times 7.45/100}{74.5} = 10^3\ mol$

Moles of Cl_2 required $= 10^3$

eq. of Cl_2 required $= 2 \times 10^3$

$2 \times 10^3 \times 96500 = 9.65 \times t$

$2 \times 10^7 = t$

Sol.16 [1]

Dissolved $\left[Zn(OH)_2\right] = \left[Zn^{+2}\right]_{aq} + \left[Zn(OH^+)_2\right]_{aq}$

$+ \left[Zn(OH_2)_{aq}\right] + \left[Zn(OH)_3^-\right] + \left[Zn(OH)_4\right]^{2-}$

Now, $\left[Zn(OH)_2\right]_{aq} = 10^{-6} M$ in saturated solution.

So, $\left[Zn(OH)\right]^+ = \dfrac{10^{-6} \times 10^{-7}}{\left[OH^-\right]} = \dfrac{10^{-13}}{\left[OH^-\right]}$

Similarly, $\left[Zn^{+2}\right] = \dfrac{10^{-17}}{\left[OH^-\right]^2}$,

$\left[Zn(OH)_3\right] = 10^{-3}\left[OH^-\right]$,

$\left[Zn(OH)_4^{2-}\right] = K_5\left[Zn(OH)_3^-\right]$

$\left[OH^-\right] = \left(10^{-2} M^{-1}\right)\left[OH^-\right]^2$

Dissolved $Zn(OH)_2 = \dfrac{10^{-17}}{\left[OH^-\right]^2} + \dfrac{10^{-13}}{\left[OH^-\right]}$

$+ 10^{-6} + 10^{-3} \times \left[OH^-\right] + 10^{-2}\left[OH^-\right]^2$

$= \dfrac{10^{-17}}{10^{-16}} + \dfrac{10^{-13}}{10^{-8}} + 10^{-6} + 10^{-3} \times 10^{-8} + 10^{-18}$

$= 10^{-1} + 10^{-5} + 10^{-6} + 10^{-11} = 10^{-1}$

$= -\log\ Zn(OH)_2(aq) = 1$

Sol.17 [6]

	$C_{12}H_{22}O_{11} + H_2O \rightarrow$	$C_6H_{12}O_6 +$	$C_6H_{12}O_6$
mol	0.0125	0	0
	$0.0125 - x$	x	x

$\Delta T_b = m_1K_b + m_2K_b + m_3K_b$

$m_1 + m_2 + m_3 = \dfrac{0.104}{0.52} = 0.2$

$$\frac{0.125 - x + x + x}{100} \times 100 = 60$$

$$x = 0.0075$$

$$mol\ \% = \frac{0.0075}{0.0125} \times 100 = 60$$

$$\frac{1}{10} th\ of\ mol\ \% = \frac{60}{10} = 6$$

Sol.18 [2]

$$2.303 \log K = -\frac{E_a}{RT} + 2.303 \log A$$

Thus, $-\frac{E_a}{2.303R} = \tan\theta = -\frac{1}{2.303}$

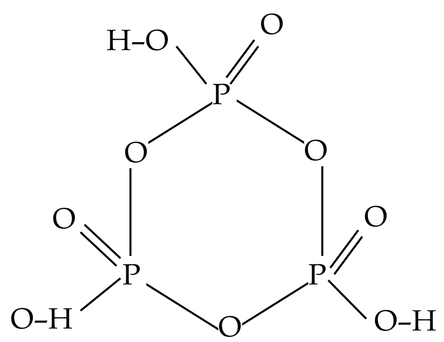
$\therefore\ E_a = R = 2\ Cal.$

Sol.19 [4]

$$\%\ of\ carbon = \frac{144}{144 + m + 35.5(10 - m)} \times 100 = 40$$

On solving $m = 4$

Sol.20 [3]



PHYSICS

Sol.21 [b, c, d]

at point 3

$$\frac{dv}{dr} = 0$$

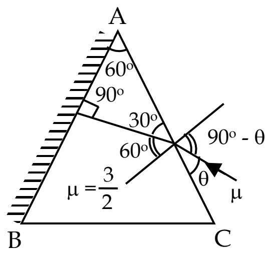
$\therefore E = 0$

Since near charge Q_2, V is –*ve* while Q_1 it is $+ve$, we can conclude $Q_2\ and\ Q_1$ are negative and positive respectively.

To the right of Q_2 the potential is $+ve$ this implies that in the entire region to the right of Q_2 the potential produced by Q_1 is greater in absolute value than the potential produced by Q_2

$\therefore |Q_1| > |Q_2|$

Sol.22 [b, c]

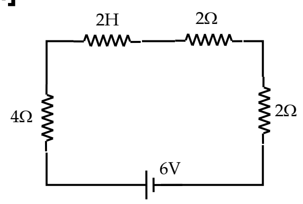


From snell's law

$$\mu \sin(90^\circ - \theta) = \frac{3}{2}\sin 60^\circ \qquad \mu \cos\theta = \frac{3\sqrt{3}}{4}$$

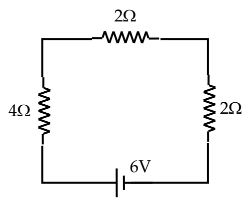
$$\theta = \cos^{-1}\left(\frac{3\sqrt{3}}{4\mu}\right)$$

Sol.23 [a, c, d]



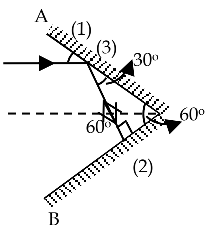
$$\tau = \frac{L}{R} = \frac{2}{8} = \frac{1}{4}\text{sec}$$

in steady state



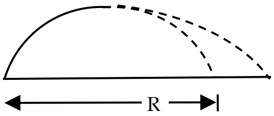
$$I = \frac{6}{8} = \frac{3}{4}\,Amp = 0.75\ Amp.$$

Sol.24 [a, d]



An mirror light fall normally. Thus light retrace and reflect at mirror *A*. Total no. of reflection is 3. Deviation of light is π .

Sol.25 [a, b, d]



By mass moment

$m_1 \times x_1 = m_2 \times x_2$

$mR = 2m \times x_2$

$x_2 = R/2$

Distance from launch point $= 3R/2$

$R = \dfrac{u^2 \sin 2\theta}{g} = 108m$

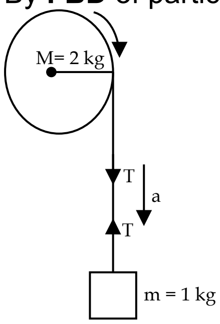
Sol.26 [b, d]

$F_{net} = \dfrac{GMm}{4R^2} - \dfrac{GMm}{18R^2}$

$$\boxed{F_{net} = \frac{7}{36}\frac{GMm}{R^2}}$$

Sol.27.[a, b, c, d]

By **FBD** of particle



$mg - T = ma$

$10 - T = a$ …..(i)

By **FBD** of disc

$TR = I \propto = L\dfrac{a}{R} \Rightarrow T = \dfrac{MR^2}{2}\dfrac{a}{R^2}$

$T = Ma/2 = a$ …..(ii)

By eq. (i) and (ii)

(a) $a = 5\ m/s^2\ \ and\ \ T = 5N\ \ and\ \ \propto = a/R = 5\ rad/s^2$

(b) For angular displacement of disc :

$\theta = \omega t + 1/2\ \propto t^2$

(c) Work done by torque

$$= \int \tau \, d\theta = \tau \int d\theta = 5 \times 40 = 200 \ J$$

(d) $\Delta K.E. = \Delta w = 200 \ J$

$$K_2 - K_1 = 200 \ J$$

Sol.28 [a, b, c]

(a) Heat required is

$$Q = nCp\Delta T = n(Cv + R)\Delta T$$

$$= \frac{1000}{28}[5+2] \times 120 = 30 \times 10^3 \ cal$$

$$Q = 30 \ Kcal$$

(b) The increase in the internal energy is

$$\Delta U = nCv\Delta T = \frac{1000}{28} \times 5 \times 120$$

= 21 Kcal

(c) $Q = nCv\Delta T + P\Delta V$

for constant volume $\Delta V = 0$

$$Q = nCv\Delta T = 21 \ Kcal$$

(d) external work done is

$$W = Q - \Delta U = 8.6 \ KCal$$

Sol.29 [a, c]

Maximum speed of any point on the string $= a\omega$

$$= a(2\pi f)$$

$\therefore = \frac{v}{10} = \frac{10}{10} = 1$ (Given : v = 10 m/s)

$\therefore 2\pi a f = 1$ $\therefore f = \frac{1}{2\pi a}$

$a = 10^{-3} m$ (Given)

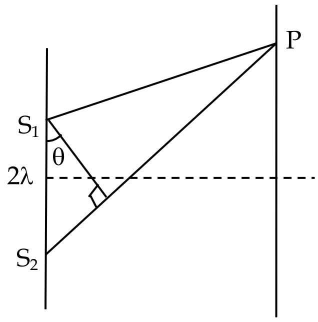
$$\therefore f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} Hz$$

Speed of wave, $v = f\lambda$

$$\therefore (10 \, m/s) = \left(\frac{10^3}{2\pi} s^{-1} \right) \lambda$$

$$\therefore \lambda = 2\pi \times 10^{-2} m$$

Sol.30 [a, d]



$\delta = \frac{\pi}{2} - \frac{2\pi}{\lambda}(2\lambda \sin\theta) \Rightarrow \delta = \frac{\pi}{2} - 4\pi \sin\theta$

For maxima, $\delta = n\pi$

Where $n = 0, \pm 1, \pm 2$ ......

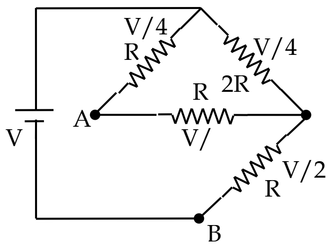
$\frac{\pi}{2} - 4\pi \sin\theta = n\pi$

$\sin\theta = \frac{\frac{1}{2} - n}{4}$ $\quad n = 0,\ \sin\theta = \frac{1}{8}$

$n = \pm 1, \sin\theta = -\frac{1}{8}, +\frac{3}{8}$ $\quad n = \pm 2, \sin\theta = -\frac{3}{8}, \frac{5}{8}$

$n = \pm 3, \sin\theta = -\frac{5}{8}, \frac{7}{8}$ $\quad n = 4, \sin\theta = -\frac{7}{8}$

Sol.31 [4]



$V_A - V_B = \frac{V}{4} + \frac{V}{2} \Rightarrow \frac{3V}{4}$

$\therefore U_{stored} = \frac{1}{2} \times 10 \times 10^{-3} \times \left(\frac{3V}{4}\right)^2$

$\Rightarrow \frac{4.5}{10} = \frac{5 \times 10^{-3} \times 9 \times V^2}{16}$

$V^2 = \frac{16}{10 \times 10^{-3}} \Rightarrow 1600$

V = 40 volt

Sol.32 [3]

Range will become twice of velocity of efflux becomes twice. Now as

$v = \sqrt{2gh}$

Therefore, *h* should become 4 times or 40 m Thus, an extra pressure equivalent to 30 m of water should be applied.

$1 \ atm = 0.76 \times 13.6 \ m$ of water

= 10.336 *m* of water

$30 \ m \ of \ water \ \approx 3.0 \ atm$

Sol.33 [3]

$$R = \frac{\ell|}{KA}$$

When heat is transferred from first vessel to second, temperature of first vessel decreases while that of second vessel increases. Due to both there reasons, difference between temperature of vessels decreases.

Let at an instant *t*, the temperature difference between two vessels be θ.

$$H = \frac{\theta}{R} = \frac{KA\theta}{\ell}$$

$$dQ = Hdt = \frac{KA\theta}{\ell} dt \qquad \text{....(i)}$$

Since gases are contained in two vessels, therefore, processes on gases in two vessels are isochoric.

Hence, decrease in temperature of gas in first vessel,

$$\Delta\theta_1 = \frac{dQ}{nC_v} = \frac{dQ}{2 \times \frac{5R}{2}} = \frac{dQ}{5R}$$

Increase in temperature of gas in second vessel is

$$\Delta\theta_2 = \frac{dQ}{4 \times \frac{3R}{2}} = \frac{dQ}{6R}$$

$\therefore$ Decrease in temperature difference

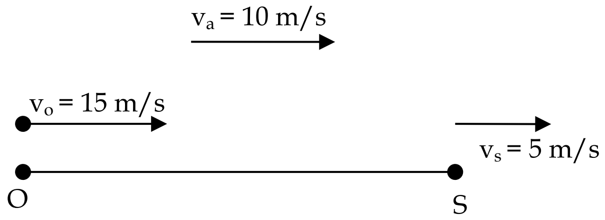
$$(-d\theta) = \Delta\theta_1 + \Delta\theta_2$$

$$-d\theta = \frac{dQ}{R} \times \frac{11}{30}$$

or $-\int_{50}^{25} \frac{d\theta}{\theta} = \frac{KA \times 11}{30\ell R} \int_0^t dt$

t = 3 seconds.

Sol.34 [5]



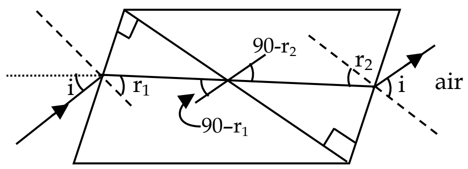
$\therefore v_{observer,\ air} = 5\ m/s$

$v_{source,\ air} = -5\ m/s$

$\Rightarrow v = \frac{330+5}{330-5} \times 325 = 335 Hz$

$\therefore v - 330 = 5\ Hz$

Sol.35 [5]



$\sqrt{3}\ \sin\ i = \sqrt{3}\ \sin\ r_1$ ....(1)

$\sqrt{3}\ \sin\ (90 - r_1) = \sqrt{2}\ \sin(90 - r_2)$

$\sqrt{3}\ \cos\ r_1 = \sqrt{2}\ \cos\ r_2$ ....(2)

$\sin\ i = \sqrt{2}\ \sin\ r_2$ ....(3)

$i = r_1$

$3\cos^2\ r_1 + \sin^2 i = 2$

$2\cos^2\ r_1 + \cos^2\ r_1 + \sin^2\ r_1 = 2$

$\cos^2\ r_1 = \frac{1}{2}$

$r_1 = 45^\circ\ \therefore\ i = 45^\circ$

Sol.36 [4]

${}^{23}_{10}Ne \rightarrow {}^{23}_{11}Na + e^- + \bar{v}$

$Q = \left[m\left({}^{23}Ne\right) - m\left({}^{23}Na\right)\right] \times 931.5\ MeV$

$A = 4.375\ MeV = 4.4\ MeV$

$Q \simeq 4\ MeV$

$Q = KE_y + KE_e + E\ \bar{v}$

KE_y is very very small

$A \approx KE_e + E\,\bar{v}$

when KE_e is maximum $E\,\bar{v}$ is negligible

$KE_e \simeq Q = 4\ MeV$

Sol.37 [5]

The energy of the electron in the n^{th} state of He^+ ion of atomic number *Z* is given by

$E_n = -(13.6)\ eV \dfrac{Z^2}{n^2}\ for\ H^+\ ion\ Z = 2$.

Therefore

$E_n = -\dfrac{(13.6\ eV)\times(2)^2}{n^2} = -\dfrac{54.4}{n^2} eV$

The energies $E_1\ and\ E_2$ of the two emitted photons in *eV* are

$E_1 = \dfrac{12431}{1085} eV = 11.4\ eV$

and $E_2 = \dfrac{12431}{304}\ eV = 40.9\ eV$

Thus total energy $E = E_1 + E_2 = 11.4 + 40.9$

$= 52.3 eV$

Let *n* be the principle quantum number of exited state.

Now we have for the transition from $n = n\ to\ n = 1$

$E = -(54.4) eV \left(\dfrac{1}{1^2} - \dfrac{1}{n^2} \right)$

But $E = 52.3\ eV$. Therefore

$52.3\ eV = 54.4\ eV \times \left(\dfrac{1}{1^2} - \dfrac{1}{n^2} \right)$

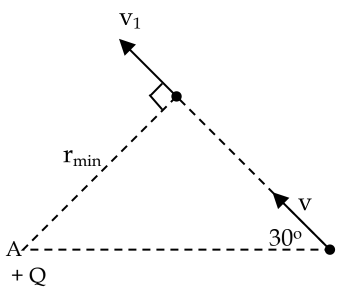
or $1 - \dfrac{1}{n^2} = \dfrac{52.3}{54.4} = 0.96$

Which gives $n^2 = 25\ or\ n = 5$

The energy of the incident electron =100 eV (given). The energy supplied to $He^+\ ion = 52.3\ eV$

Therefore, the energy of the electrons left after the collision $= 100 - 52.3 = 47.7\ eV$

Sol.38 [8]



Angular momentum about A is conserved as

$\tau_{external} = 0$

$mv \sin 30^\circ \times R = v_1 \times m \times r_{\min}$

$$\frac{Rv}{2}=\frac{1}{\sqrt{3}}\times r_{\min}$$

$$r_{\min}=\frac{\sqrt{3}R}{2}=\frac{1.713\times 10}{2}=8.665=8\,cm$$

Sol.39 [9]

At y according to Kirchoff's junction law

$$\frac{y-x}{2}+\frac{y-x-100}{2}+\frac{y-50}{2}+\frac{y}{2}+\frac{y-50}{2}=0$$

$5y-2x=200$ ….(1)

Similarly at x

$i=\frac{50-x}{2}+\frac{y-x}{2}$ …..(2)

$at\quad x+100$

$i=\frac{x+100-50}{2}+\frac{x+100-y}{2}$ ….(3)

We get $y-2x=50$ ….(4)

From (1) and (4)

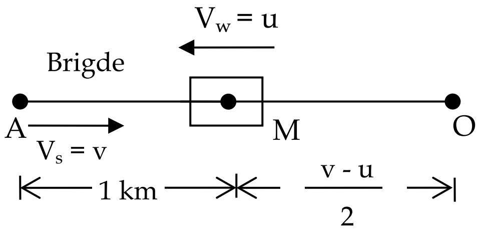
$y=37.5\ V$

So current through R is 18.75 A.

Sol.40 [1]

Let $V_w=u$ & $U_{sw}=v$

Time taken by swimmer to go from M to O and O to B = time taken by float to reach B from M.



$$=\frac{1}{2}+\frac{1+\frac{v-u}{2}}{v+u}=\frac{1}{u}$$

$$\Rightarrow \frac{1}{2}+\frac{2+v-u}{2(v+u)}=\frac{1}{u}$$

$$\Rightarrow \frac{(v+u+2+v-u)}{2(v+u)}=\frac{1}{u}$$

$$\Rightarrow (2v+2)u=2(v+u)$$

$$\Rightarrow 2vu+2u=2v+2u$$

$$u=1\ km/hr$$

MATHEMATICS

Section – I [Q.1 to Q. 10]

Sol.41 [a, b, d]

$$f(x)=\begin{cases}\dfrac{1}{x+1} & , \quad 0\le x<1\\ \dfrac{2}{x} & , \quad 1\le x<2\\ \dfrac{3}{x-1} & , \quad 2\le x<\dfrac{5}{2}\end{cases}$$

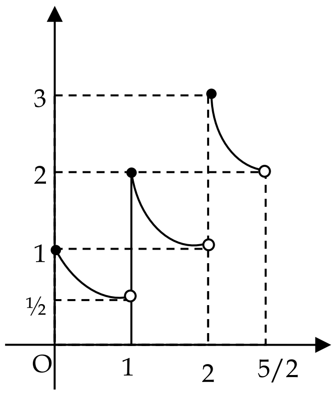
Clearly $f(x)$ is discontinuous and bijective function

$$\lim_{x\to 1^-} f(x)=\frac{1}{2}$$

$$\lim_{x\to 1^+} f(x)=2$$

$$\min\left(\lim_{x\to 1^-} f(x), \lim_{x\to 1^+} f(x)\right)=\frac{1}{2}\ne f(1)$$

$$Max\ (1,\ 2)=2=f(1)$$



Sol.42 [b, c]

From hyperbola and conjugate hyperbola

$$\frac{1}{e^2}+\frac{1}{(f(e))^2}=1$$

$$\Rightarrow f(e)=\frac{e}{\sqrt{\ }}$$

$$f(f(f(e)))=\frac{e}{\sqrt{e^2-1}}$$

$$\underbrace{fff\,.........f(e)}_{n\ times}=\begin{cases} e, & n\ is\ even\\ \dfrac{e}{\sqrt{e^2-1}}, & n\ is\ odd\end{cases}$$

n is even

$$\int_1^3 \underbrace{fff......f(e)}_{n\ times}\, de = \int_1^3 e\ de = \left.\frac{e^2}{2}\right|_1^3 = 4$$

n is odd

$$\int_1^3 \underbrace{fff........f(e)}_{n\ times}\, de = \int_1^3 \frac{e}{\sqrt{e^2-1}}\, de = \left.\sqrt{e^2-1}\right|_1^3$$

$$= \sqrt{8} - 0 = 2\sqrt{2}$$

Sol.43 [a, c]

Equation of tangent at $P(4\cos\theta, 2\sin\theta)$ is

$x\cos\theta + 2y\sin\theta = 4$

It pass through (4, 2)

$\therefore \sin\theta + \cos\theta = 1$

$\theta = 0,\ \frac{\pi}{2}$

Sol.44 [a, c, d]

$$\sin\left(x\frac{dy}{dx} - y\right) = \frac{dy}{dx}$$

$$x\frac{dy}{dx} - y = \sin^{-1}\frac{dy}{dx} \qquad \text{....(i)}$$

Again diff.

$$\frac{dy}{dx} + \frac{xd^2y}{dx^2} - \frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{dy}{dx}\right)^2}}\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2}\left(x - \frac{1}{\sqrt{1-\left(\frac{dy}{dx}\right)^2}}\right) = 0$$

$$\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{dy}{dx} = c$$

From equation (i)

$cx - y = \sin^{-1} c$

$c = 0,\ \ y = 0$

Or $\quad x = \dfrac{1}{\sqrt{1-\left(\frac{dy}{dx}\right)^2}}$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = 1 - \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2-1}}{x}$$

From equation (i)

$$\frac{x\sqrt{x^2-1}}{x} - y = \sin^{-1}\frac{\sqrt{x^2-1}}{x}$$

$$y = \sqrt{x^2-1} - \sin^{-1}\frac{\sqrt{x^2-1}}{x}$$

Sol.45 [b, d]

$$f(x) = x^2 + ax^2 + bx^3\Big| = (a+1)x^2 + bx^3$$

$$a = \int_{-1}^{1} t\ f(t)\ dt = \int_{-1}^{1} (a+1)t^3 + bt^4\ dt$$

$$= 2b\int_{0}^{1} t^4\ dt$$

$$a = \frac{2b}{5} \qquad \text{….(1)}$$

$$b = \int_{-1}^{1} f(t)\ dt = \int_{-1}^{1} (a+1)t^2 + bt^3\ dt$$

$$= \frac{(a+1)t^3}{3}\Bigg|_{-1}^{1}$$

$$b = (a+1)\frac{2}{3} \qquad \text{….(2)}$$

$$a = \frac{4}{11} \qquad b = \frac{10}{11}$$

$$f(x) = \frac{15x^2}{11} + \frac{10x^3}{11} \qquad f(1) = \frac{15}{11} + \frac{10}{11} = \frac{25}{11}$$

$$f(-1) = \frac{15}{11} - \frac{10}{11} = \frac{5}{11} \qquad f(1) + f(-1) = \frac{30}{11}$$

$$f(1) - f(-1) = \frac{20}{11}$$

Sol.46 [a, b, c, d]

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Angle between $\vec{b}$ *and* $\vec{c}$

= angle between $\vec{a}$ *and* $\vec{c}$

$$\Rightarrow \vec{b}\ .\ \vec{c} = \vec{a}\ .\ \vec{c} = \cos\alpha$$

$$\left(\because |\vec{a}| = |\vec{b}| = |\vec{c}| = 1\right)$$

also $\vec{a} \perp \vec{b} \Rightarrow \vec{a}\ .\ \vec{b} = 0$

so $\vec{c} = \ell\vec{a} + m\vec{b} + n\left(\vec{a}\times\vec{b}\right)$

$\therefore\ \vec{a}\ .\ \vec{c} = \ell = \cos\alpha,\ \vec{b}\ .\ \vec{c} = \cos\alpha = m$

Now $|\vec{c}| = 1 \Rightarrow \vec{c}\ .\ \vec{c} = 1 = 2\ell^2 + n^2\left|\vec{a}\times\vec{b}\right|^2$

$\Rightarrow m = \ell = \cos\alpha$

$\Rightarrow 1 = 2\ell^2 + n^2$

$\Rightarrow n^2 = 1 - 2\ell^2 = 1 - 2\cos^2\alpha = -\cos 2\alpha$

$\Rightarrow \ell^2 = m^2 = \dfrac{1-n^2}{2} = \dfrac{1+\cos 2\alpha}{2}$

Hence all are correct.

Sol.47 [b, c]

$x^2 + 400 = y^2$ ….(1)

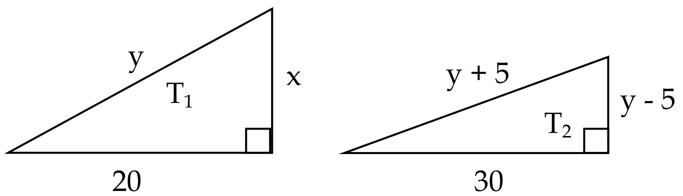
& $900 + (x-5)^2 = (y+5)^2$

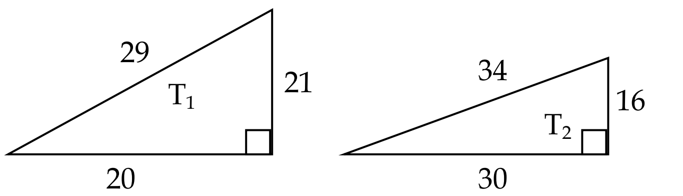
$\Rightarrow 900 + x^2 - 10x = y^2 + 10y$

$\Rightarrow 900 + x^2 - y^2 = 10(x+y)$

$\Rightarrow 500 = 10(x+y)$

$\Rightarrow x + y = 50$ ….(2)





Form (1) & (2)

$y - x = 8$

From (2) & (3) ….(3)

So, $y = 29\ \&\ x = 21$

$\therefore \Delta_1 = 210\ \ \Delta_2 = 240$

$\therefore \dfrac{\Delta_2}{\Delta_1} = \dfrac{8}{7} \Rightarrow 8\Delta_1 = 7\Delta_2$

Also, $P_1 = \dfrac{\Delta_1}{S} = \dfrac{210}{35} = 6$

& $P_2 = \dfrac{\Delta_2}{S} = \dfrac{240}{40} = 6$

Sol.48 [a, b, c]

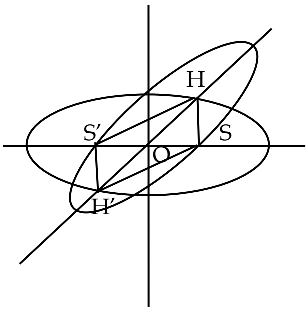
Clearly *O* is the mid point of *SS'* and *HH'*

$\Rightarrow$ diagonals of quadrilateral HSH'S' bisect each other so, it is a parallelogram .

Let $H'OH = 2r \Rightarrow OH = r = ae_2$

H lies on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (suppose)

$\therefore \frac{r^2\cos^2\theta}{a^2}+\frac{r^2\sin^2\theta}{b^2}=1$



$$e_2^2\cos^2\theta+\frac{e_2^2\sin^2\theta}{1-e_1^2}=1\left(\because b^2=a^2\left(1-e_1^2\right)\right)$$

$$e_2^2\cos^2\theta-\frac{e_2^2\cos^2\theta}{1-e_1^2}=1-\frac{e_2^2}{1-e_1^2}$$

$$\Rightarrow \cos^2\theta=\frac{1}{e_1^2}+\frac{1}{e_2^2}-\frac{1}{e_1^2e_2^2}$$

If $\theta = 90^\circ$

$$\frac{e_1^2+e_2^2}{e_1^2e_2^2}=\frac{1}{e_1^2e_2^2}=e_1^2+e_2^2=1$$

Sol.49 [c, d]

$$\lim_{n\to\infty}\tan(1/n)\ln(1/n)$$

$$=\lim_{n\to\infty}\frac{\tan(1/n)}{(1/n)}.\frac{\ln(1/n)}{n}$$

$$=-\lim_{n\to\infty}\frac{\tan(1/n)}{(1/n)}.\frac{\ln(n)}{n}$$

$$=-1.\lim_{n\to\infty}\frac{1/n}{1}=0$$

Then, $f(x)=e^0=1$

$$\therefore \int\frac{f(x)}{\sqrt[3]{\left(\sin^{11}x\cos x\right)}}dx=\int\frac{1}{\sin^{11/3}x\cos^{1/3}x}dx$$

$$=\int\sin^{-11/3}x.\cos^{-1/3}x\,dx$$

$$=\int(\tan x)^{-11/3}\cos^{-4}x dx=\int(\tan x)^{-11/3}.\cos^{-4}x\,dx$$

$$= \int (\tan x)^{-11/3}.(1+\tan^2 x).\sec^2 x\, dx$$

$$= \frac{(\tan x)^{-\frac{11}{3}+1}}{\left(\frac{-11}{3}+1\right)} + \frac{(\tan x)^{-2/3}}{(-2/3)} + c$$

$$= -\frac{3}{8}(\tan x)^{-8/3} - \frac{3}{2}(\tan x)^{-2/3} + c$$

$$\therefore g(x) = -\frac{3}{8}(\tan x)^{-8/3} - \frac{3}{2}(\tan x)^{-2/3}$$

$$\therefore g(\pi/4) = -\frac{3}{8} - \frac{3}{2} = -\frac{15}{8}$$

and $g(x)$ is non differentiable at $\tan x = 0 \ or \ x = n\pi, n \in I$

Sol.50 [b, c]

Given $f_6(x) = \frac{x-1}{x}$ ....(1)

$f_6(f_m(x)) = f_4(x) = \frac{1}{1-x}$ (given)

$\therefore \ f_6(f_m(x)) = \frac{f_m(x)-1}{f_m(x)} = \frac{1}{1-x}$

(using given relation $f_6(x) = \frac{x-1}{x}$)

Put $f_m(x) = k$

$$\frac{k-1}{k} = \frac{1}{1-x}$$

$$k = kx - 1 + x = k \Rightarrow k = \frac{x-1}{x}$$

$$\Rightarrow f_m(x) = \frac{x-1}{x} = f_6(x) \Rightarrow m = 6$$

Again $f_n(f_4(x)) = f_3(x) = \frac{1}{x}$

$$f_n\left(\frac{1}{1-x}\right) = \frac{1}{x}; \ let \ \frac{1}{1-x} = t$$

$$\Rightarrow t - tx = 1 \Rightarrow x = \frac{t-1}{t}$$

$$\therefore \ f_n(t) = \frac{t}{t-1} \Rightarrow f_n(x) = \frac{x}{x-1} = f_5(x)$$

Hence $n = 5$

Sol.51 [2]

Volume of the parallelepiped formed by $\vec{a}', \vec{b}', \vec{c}'$ is 4

$\therefore$ Volume of the parallelopiped formed by $\vec{a}, \vec{b}, \vec{c}$ *is* $\frac{1}{4}$

$$\vec{b} \times \vec{c} = \frac{(\vec{c}' \times \vec{a}') \times \vec{c}'}{4} = \frac{1}{4}\vec{a}'$$

$$\therefore \left|\vec{b} \times \vec{c}\right| = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$$

$\therefore$ length of altitude $= \frac{1}{4} \times 2\sqrt{2} = \frac{1}{\sqrt{2}}$

Sol.52 [1]

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

If $AB = BA,\ then\ a + 2c = a + 3b$

$\Rightarrow 2c = 3b \Rightarrow b \neq 0$

$b + 2d = 2a + 4b \quad \Rightarrow 2d - 2a = 3b$

$$\frac{d-a}{3b-c} = \frac{\frac{3}{2}b}{3b - \frac{3}{2}b} = 1$$

Sol.53 [4] $\frac{E(1) + E(2) + E(3) + ... + E(100)}{100}$

$$= \frac{E(010203.....99.100)}{100}$$

$$= \frac{20(2+4+6+8)}{100} = \frac{400}{100} = 4$$

Sol.54 [5]

Highest power of *x* in numerator & denominator is 2010

$$\therefore \lim_{x \to \infty} \frac{x^{2010} \sum_{r=1}^{10} \left(1 + \frac{r}{x}\right)^{2010}}{x^{1006}\left(1 + \frac{1}{x^{1006}}\right) x^{1004}\left(2 + \frac{1}{x^{1004}}\right)}$$

$$= \frac{\sum_{r=1}^{10}(1)}{(1+0)(2+1)} = \frac{10}{2} = 5$$

Sol.55 [3]

$$\lim_{x\to 0}\frac{\sin x^4 - x^4\cos x^4 + x^{20}}{x^4\left(e^{2x^4}-1-2x^4\right)}$$

$$=\lim_{t\to 0}\frac{\sin t - t\cos t + t^5}{t\left(e^{2t}-1-2t\right)}$$

$$=\lim_{t\to 0}\frac{t-\frac{t^3}{3!}+\frac{t^5}{5!}.....-t\left(1-\frac{t^2}{2!}+\frac{t^4}{4!}.....\right)+t^5}{t\left(1+2t+\frac{4t^2}{2!}+\frac{8t^3}{3!}+\frac{16t^4}{4!}+.....-1-2t\right)}$$

$$=\lim_{t\to 0}\frac{-\frac{t^3}{6}+\frac{t^3}{2}+\frac{t^5}{5!}-\frac{t^5}{4!}+....+t^5}{2t^3+\frac{8t^4}{3!}+.....}$$

$$=\frac{-\frac{1}{6}+\frac{1}{2}}{2}=\frac{-1+3}{12}=\frac{1}{6}$$

Sol.56 [2]

Urn A (9 R, 11 W) ; Urn B (12 R, 3 W)

E : event of drawing a red ball;

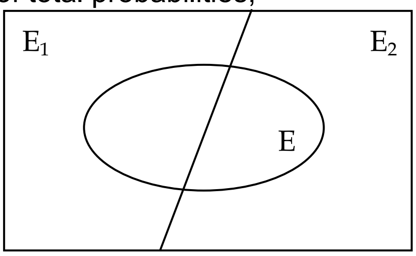
$E_1 = 1\ or\ 2$ on die

$E_2 = 3, 4, 5, 6$ on die

$E = (E\cap E_1)+(E\cap E_2)$

$P(E)=P(E_1).\,P(E/E_1)+P(E_2)\,(E/E_2)$

Using the law of total probabilities,



$$P(red\ \ ball)=\frac{2}{6}.\frac{9}{20}+\frac{4}{6}.\frac{12}{15}=\frac{41}{60}$$

Sol.57 [1]

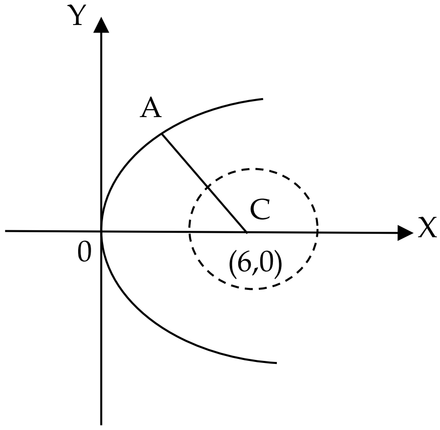
$f(1)=-6$

For maximum at $x = 1$

$\lim_{x\to 1^-} f(x) = \tan^{-1}\alpha - 5 < -6$

$\Leftrightarrow \tan^{-1}\alpha < -1 \quad \Leftrightarrow \alpha < -\tan 1$

Sol.58 [4]



Any normal of parabola $y^2 = 4x$ is

$y = -tx + 2t + t^3$

If it pass throught (6,0), then

$-6t + 2t + t^3 = 0$

$\Rightarrow t = 0\ ;\ t^2 = 4$

$\therefore A\ (4, 4))$

$\therefore$ for no common tangent $AC > r$

$\Rightarrow \sqrt{20} > r \Rightarrow r < \sqrt{20}$

Sol.59 [3]

$y = \frac{e^x - e^{-x}}{2} \Rightarrow e^{2x} - 1 = 2ye^x$

Therefore, $t^2 - 2yt - 1 = 0, t = e^x$

$\Rightarrow t = \frac{2y \pm \sqrt{4y^2 + 4}}{2} = y \pm \sqrt{y^2 + 1}$

$\Rightarrow x = \log\left(y + \sqrt{y^2 + 1}\right)\left(Since\ e^x > 0\right)$

$\therefore f^{-1}(x) = g(x) = \log\left(x + \sqrt{x^2 + 1}\right)$

$g\left(\frac{e^{1002} - 1}{2e^{501}}\right) = \log\left(\frac{e^{1002} - 1}{2e^{501}} + \frac{e^{1002} + 1}{2e^{501}}\right)$

$= \log\ e^{501} = 501$

Sol.60 [4]

Equation of plane $\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$

$\Rightarrow 11x - y - 3z = 28$

$m = -1,\ n = -3$

$|m+n| = 4$

PAPER – 2

CHEMISTRY

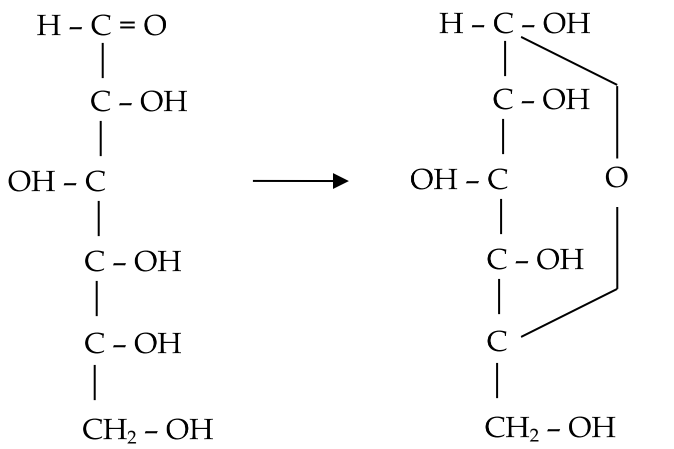
Sol.1 **(d)**

Phenolic structure [Stabilised by Resonance]

Sol.2 **(d)**

For benzyne mechanism at least 1 β -hydrogen is necessary.

Sol.3 **(c)**



(Five hydroxyl groups) (Five hydroxyl groups)

Sol.4 **(c)**

It is given by 1° amide

Sol.5 **(b)**

$ClO_3^- + 2H_2 + 4e^- \rightarrow ClO^- + 4OH^-;\ \Delta G_1^\circ$

$ClO^- + H_2O + e^- \rightarrow \frac{1}{2}Cl_2 + 2OH^-;\ \Delta G_2^\circ$

$\frac{1}{2}Cl_2 + e^- \rightarrow Cl^- \quad ;\ \Delta G_3^\circ$

$$ClO_3^- + 3H_2O + 3e \rightarrow Cl^- + 6OH^-;\ \Delta G^\circ$$

$$\therefore \Delta G^\circ = \Delta G_1{}^\circ + \Delta G_2{}^\circ + \Delta G_3{}^\circ$$

$$-6FE^\circ = -4F \times 0.54 - 1F \times 0.45 - 1F \times 1.07$$

$$\therefore E^\circ = +\frac{3.68}{6} = +0.61\ V$$

Sol.6 (d)

$$n_{H^+} = 10^{-2} V$$

$$n_{OH^-} = 10^{-2} V$$

so, solution is neutral & $pH = 6$

Sol.7 (c)

The loss in weight should be proportional to vapour pressure above that solution :

So, $P_{S_A} \propto 2gm$

$P_{S_n} \propto 1.5\ gm$

$P_{S_C} \propto 2.5\,gm$

So, maximum vapour pressure is above *C* solution hence, it is having minimum lower and hence minimum mole fraction (hence minimum number of moles of solute) So max, molar mass of substance.

Sol.8 (b)

$r_2 = k_2 [A]_2^1\ [B]_2^1$ for a certain run

$r_1 = k_1 [A]_1^1 [B]_1^1$ for a previous run

$$\frac{r_2}{r_1} = \frac{k_2}{k_1} \frac{[A]_2}{[A]_1} \frac{[B]_2}{[B]_1}$$

Substituting the given information

$$1.5 = 2^{\left(\frac{t_2 - 27}{10}\right)} \times \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow 6 = 2^{\frac{t_2 - 27}{10}}$$

$$\Rightarrow \frac{t_2 - 27}{10} \ell n 2 = \ell n\, 6$$

$$\Rightarrow \frac{t_2 - 27}{10} = \frac{\ell n 6}{\ell n 2}$$

$$\Rightarrow \frac{t_2 - 27}{10} = 2.585$$

$$\Rightarrow t_2 = 52.85^\circ C = 53^\circ C$$

Sol.9 (c)

$\Delta G^{\circ} = RT \ \ln \ K_{eq}$

Or, $-1743 = -8.3 \times 300 \times \ \ln \ keq$

$k_{eq} = 2 = k_{eq}$ for reaction $2A \rightarrow B + C$ from given data.]

Sol.10 (c)

(a), (b) M. O. for

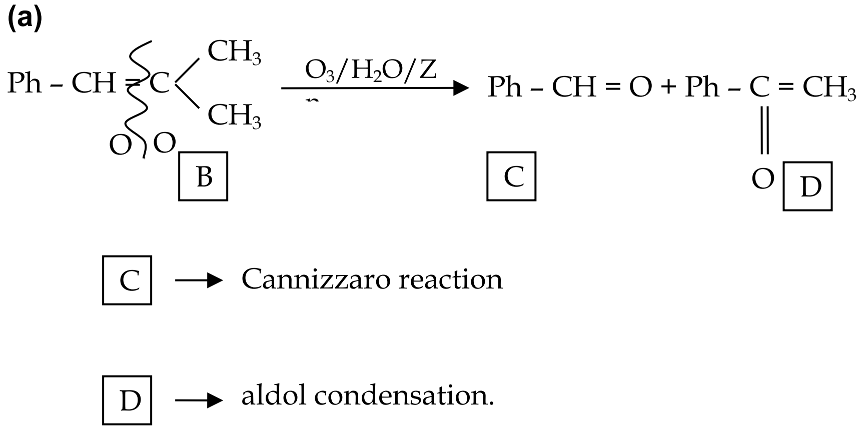
$C = \sigma 1s^2 < \overset{*}{\sigma} 1s^2 < \sigma 2s^2 < \overset{*}{\sigma} 2s^2$

$< \underbrace{\pi 2p_y^2 = \pi 2p_z^2}_{HOMO} < \underbrace{\sigma 2p_x}_{LOMO}$, two π molecular orbitals are involved in bonding.

(c) It is isoelectronic with N_2 and has one sigma and two pie-bonds.

(d) In both, all electrons are paired so diamagnetic.

Sol.11 (a)



Sol.12 (a)

Sol.13 (a)

At anode

$H^+ + e^- \rightarrow \frac{1}{2} H_{2(g)}$

$n_{e^-} = n_{H^+} = 0.165$

Sol.14 (c)

At cathode

$n_{e^-} = n_{OH^-} = 0.165$

$So,$	$H_2PO_4^-$ +	OH^- $\rightarrow$	HPO_4^{2-} +	H_2O
$t = 0$	0.33	0.165	0.33	
(after reaction)	$(0.33 - 0.165)$	0	$(0.33 + 0.165)$	

Therefore $pH = 7.2 + \log_{10} \frac{0.469}{0.165} = 7.65$

Sol.15 (c)

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C \xrightarrow{\lambda_3} D$$

Since $\lambda_1 >> \lambda_2 << \lambda_3$

We can assume that all the '*A*' has been converted in '*B*' in small duration Number of moles of *C* formed = number of moles of '*B*' dissociated

$$\Delta B = \lambda_2 \; N \; t = \frac{\ln 2}{6930} \times 10^{20} \times 10 = 10^{17}$$

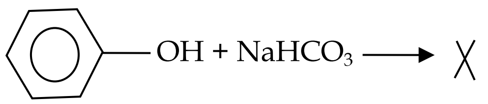
Sol. 16(b)

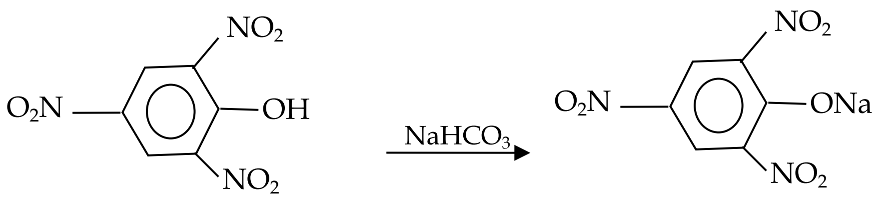
Number of nuclei of '*D*' formed = number of nuclei of '*B*' disintegrated $= \frac{1}{2} \times 10^{20}$]

Each Question from 17 to 20 has matching lists. The codes for the lists have choices (A, B, C and D) out of which ONLY ONE is correct. Match List – I with List – II and select the correct answer using the code given below the lists. +3 marks for the SINGLE CORRECT ANSWER and -1 for the INCORRECT ANSWER.

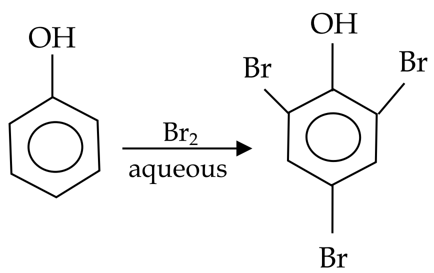
Sol.17 (a)

Sol.18 (c)





$$Phenol + neutrul \; FeCl_3 \; \rightarrow violet \;\; colour$$



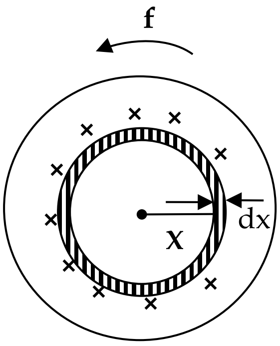
2,4,6 – Tribromo phenol (white ppt)

Sol.19 (b) Theory

Sol.20 (d) Theory

PHYSICS

Sol.21 [a]



Charge of element at ring $dq = \sigma(2\pi x dx)$

$$dB = \frac{\mu_0 dI}{2x} \Rightarrow dB = \frac{\mu_0}{2x}\frac{dq\omega}{2\pi}$$

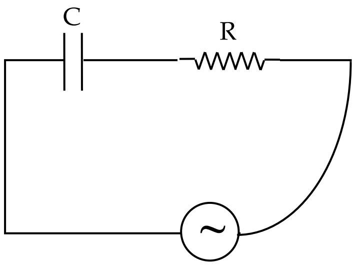
$$B = \int dB = \frac{\mu_0\omega}{4\pi}\int_0^R \frac{\sigma 2\pi\, x\, d\, x}{x}$$

$$B = \frac{\mu_0\,\omega\sigma\, R}{2} = \frac{\mu_0 2\pi\, f\, qR}{2\,\pi\, R^2} = \frac{\mu_0 q\, f}{R}$$

Sol.22 [d]

$V_L = V_C = V_R$,

$V_{source} = \sqrt{10^2 + (10-10)^2} = 10V$



$\therefore\ ix_L = ix_C = iR$

$\therefore x_L = x_C = R$

$\therefore V_c = ix_C$

$= \frac{V_{source}}{\sqrt{R^2 + x_C^2}} x_C$

$= \frac{V_{source}R}{\sqrt{R^2 + R^2}} = \frac{10V}{\sqrt{2}}$

Sol.23 [b]

$energy = power \times time = 10^6 \times 24 \times 60 \times 60$

$= 24 \times 36 \times 10^8 \ Joule$

No. of uranium atoms used $= \dfrac{24 \times 36 \times 10^8}{32 \times 10^{-12}}$

$\therefore$ required mass $= \dfrac{235}{6 \times 10^{23}} \times 27 \times 10^{20} \approx 1 \ gm$

Sol.24 (d)

$mv = (m+M)V_x \Rightarrow V_x = \dfrac{mv}{(m+M)}$

From energy conservation

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}(m+M)V_x^2 + \dfrac{1}{2}mV_y^2 + mgh_1$

$V_y^2 = v^2 - \dfrac{mv^2}{m+M} - 2gh_1$

If h_2 is the height from break off point then

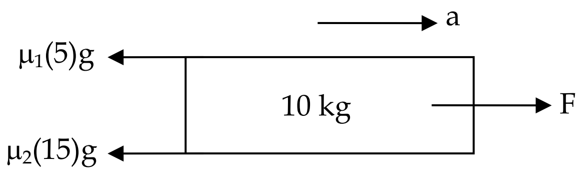
$h_2 = \dfrac{V_y^2}{2g}$

& $h = h_1 + h_2 = \dfrac{Mv^2}{2g(m+M)}$

Sol.25 [d]

$(F + 80N) - \mu_2(5kg + 10kg)g = (15 \ kg)a$

$\dfrac{F + 80 - 60}{15} = a$ ....(1)



$F - \mu_2(15) \times 10 - \mu_1(5 \times 10) = 10 \times a$

$\dfrac{F - 90}{10} = a$ ....(2)

So, we can write

$\dfrac{F + 20}{15} = \dfrac{F - 90}{10}$

$F = 310 \ N$

Sol. 26[d]

$R = u_x t + \dfrac{1}{2}a_x t^2$

$= u \cos\theta \times \dfrac{2u \sin\theta}{g} + \dfrac{1}{2}\left(\dfrac{g}{4}\right)\left(\dfrac{2u \sin\theta}{g}\right)^2$

$$= \frac{2u^2 \sin\theta\cos\theta}{g} + \frac{(u\sin\theta)^2}{2g}$$

$$= R + H, R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$H = \frac{(u\ \sin\theta)^2}{2g}$$

Sol.27 [a]

Constrain

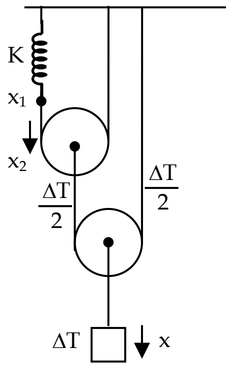
$$-x_2 + 2x = 0$$

$$x_2 = 2x$$

$$-x_1 + x_2 + x_2$$

$$x_1 = 2x_2$$

$$x_1 = 4x$$



$$2Kx_1 = \frac{\Delta T}{2} \qquad \Delta T = 4Kx_1$$

$$\Rightarrow 16\ Kx \qquad a = \frac{16Kx}{m}$$

$$\omega = G\sqrt{\frac{K}{m}} \qquad T = \frac{2\pi}{4}\sqrt{\frac{m}{K}} = \frac{\pi}{2}\sqrt{\frac{m}{K}}$$

Sol.28 [d]

$$H = \frac{Q}{t} = \frac{KA}{\ell}(\theta_1 - \theta_2)$$

$$= \frac{mL}{t} = \frac{K4\pi r^2}{\ell}(\theta_1 - \theta_2)$$

$$\frac{V\rho L}{t} = \frac{K4\pi r^2}{\ell}(\theta_1 - \theta_2)$$

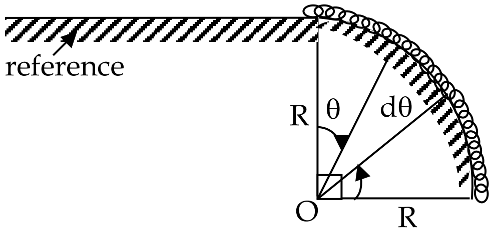
$$\frac{4}{3}\frac{\pi r^3 \rho L}{t} = \frac{K4\pi r^2}{\ell}(\theta_1 - \theta_2)$$

$$K \propto \frac{r.\ell}{t}$$

$$\frac{K_1}{K_2} = \frac{r_1}{r_2} . \frac{\ell_1}{\ell_2} \times \frac{t_2}{t_1}$$

$$\frac{K_1}{K_2} = \frac{2r}{r} \cdot \frac{\ell}{4\ell} \cdot \frac{16}{25} = \frac{8}{25}$$

Sol.29 [c]



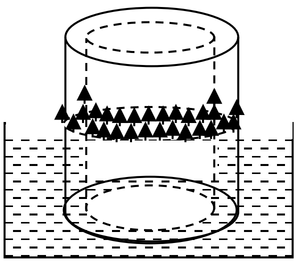
$$dU_i = -\left(\frac{m}{\ell} R d\theta\right) \times g \times R[1 - \cos\theta]$$

$$dU_i = -\frac{mgR^2}{\ell}[1 - \cos\theta] d\theta$$

$$\therefore \quad U_i = -\frac{mgR^2}{\ell}\left[\left(\frac{\ell}{R}\right) - \sin\left(\frac{\ell}{R}\right)\right]$$

and $U_f = 0 \qquad \therefore W_{ext} = \Delta U$

Sol.30 [d]



Net upward force

$= 2\pi R_2 S + 2\pi R_1 S$ contact angle $= 0^\circ$

$\therefore$ Capillary rise is given by

$$h = \frac{2\pi S(R_1 + R_2)}{\pi(R_2^2 - R_1^2)\rho g}$$

$$= \frac{2S}{(R_2 - R_1)\rho g}$$

Sol.31 [b]

Sol.32 [b]

$Emf = Bv\ell$

Here $q = CV = CBv\ell$ …(i)

$$i=\frac{dq}{dt}=C\ell B\left(\frac{dv}{dt}\right)=CB\ell a \qquad \text{...(ii)}$$

Here $F_{net}=F-F_B=F-Bi\ell$

$$ma=F-B(CB\ell a)\ell$$

$$a=\frac{F}{m+CB^2\ell^2}=const \qquad \text{...(iii)}$$

$$\frac{dv}{dt}=\frac{F}{m+CB^2\ell^2}$$

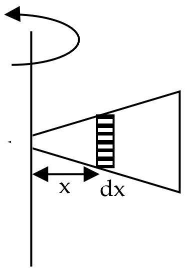
$$V=\left(\frac{F}{m+CB^2\ell^2}\right)t \qquad \text{...(iv)}$$

Equation (ii) says I is const

Equation (iv) says $V \propto t$ hence velocity $\uparrow s$

Equation (i) says with increase in velocity charge increases

Sol.33 [d]



$$\int dI=\int_0^L dm\,.x^2$$

$$=\int_0^L \lambda\, dx\,.x^2=\int_0^L K\ .\ x^3\ \ dx=\frac{KL^4}{4}$$

So choice (D) is correct and rest are wrong.

Sol.34 [d]

If *M* is mass of the square plate.

Mass of portion of each hole

$$m=\frac{M}{16R^2}\pi R^2=\frac{\pi}{16}M$$

Moment of Inertia of remaining portion

$$I=I_{square}-4I_{hole}$$

$$\frac{\pi}{16}\left[16R^2+16R^2\right]-4\left[\frac{mR^2}{2}+m\left(\sqrt{2}R\right)^2\right]I$$

$$=\left(\frac{8}{3}-\frac{10\pi}{16}\right)MR^2$$

Sol.35 [a]

The force exerted by the gas on the piston at the moment when compression of the spring is *x*, is given by

$F = P_0 A + kx$

where $P_0 = 100\ k\ Pa =$ atomospheric pressure $A = 20\,cm^2 =$ Area of cross-section of the piston and $k = 200\ N/m =$ spring constant. Hence, work done by the gas as the piston moves through $\ell = 10\,cm,$ is given by :

$$W = \int_0^{\ell} F\ dx = \int_0^{\ell} (P_0 A + kx)\ dx = P_0 A\,\ell + \frac{1}{2} k\,\ell^2$$

$$= (100 \times 103\ N/m^2) \times (20 - 10^{-4} m^2) \times (10 \times 10^{-2} m)$$

$$+ \frac{1}{2} \times (200\,N/m) \times (100 \times 10^{-4} m^2)$$

$$= 20J + 1J = 21J$$

Sol.36 [c]

The internal energy is : $U = \left(\frac{3}{2}\right) nRT$

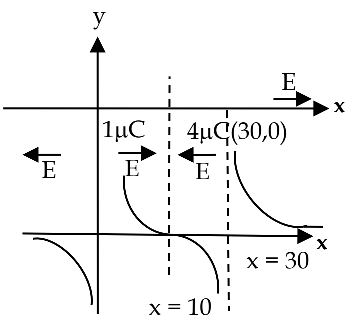
$$\therefore dU = \left(\frac{3}{2}\right) nR\Delta T$$

$$= 1.5 \times (2.0\ mol) \times (8.3\,J/mol - K) \times (31K) = 772J$$

Hence, according to first law of thermodynamics,

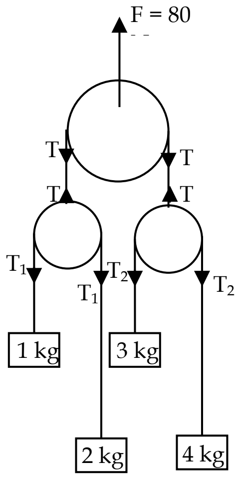
$$\Delta Q + \Delta U + \Delta W = 772\,J + 21\,J = 793\,J$$

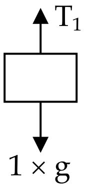
Sol.37 [a]



Electric potential decreases along electric field vector.

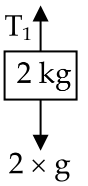
Sol.38 [b]





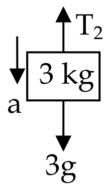
$T_1 - 1 \times g = 1 \times a$

$a = 10\ m/s^2$



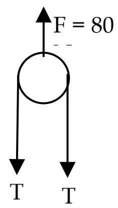
$T_1 - 20\ N = 2 \times a$

$a = 0$

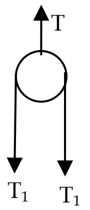


$30N - 20\ N = 3 \times a$

$a = 10/3$



80 N = 2T
T = 40 N



40 N = $2T_1$

Sol.39 [c]

$$S = \frac{1}{2} \times 2 \times 16 = 16\,m$$

$$|Wg| = mg\ S =$$

$$W_N = m(g+a)\cos^2\theta\ .\ S$$

$$W_f = m(g+a)\sin^2\theta\ .\ S$$

Sol.40 [c]

(1) $10 \times 1(\theta - 30) = 10 \times 1(70 - \theta) \Rightarrow 50^\circ C \to S$

(2) $10 \times 80 + 10 \times 1 \times (\theta - 0) = 10 \times 1 \times (85 - \theta)$

$\Rightarrow 2\theta = 5^\circ C \Rightarrow \theta = 2.5^\circ C \to P$

(3) $\theta < 80^\circ C$, so ice will not melt completely $\to Q$

SOLUTION - PRACTICE TEST - 2

(4) $m_s > \frac{m_i}{3}$, so steam will not condense completely $\to R$

MATHEMATICS

Sol.41 (b)

Since equation $5x^2 + 12x + 13 = 0$ has imaginary roots.

Hence equations $ax^2 + bx + c = 0$ & $5x^2 + 12x + 13 = 0$ have both common roots

$\therefore \frac{a}{5} = \frac{b}{12} = \frac{c}{13} = k \ (say)$

$a = 5k,\ b = 12k,\ c = 13k$

$\Rightarrow a^2 + b^2 = 25k^2 + 144k^2 = 169k^2 = (13k)^2 = c^2$

$\Rightarrow a^2 + b^2 = c^2$

$\Rightarrow ABC$ is right angled triangle

Sol.42 (c)

For the given system to have a non-trivial solution, we must have

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & k & 3 \\ 0 & -2k & -11 \\ 0 & 3-2k & -10 \end{vmatrix} = 0$$

[Applying $R_2 \to R_2 - 3R_1, R_3 \to R_3 - 2R_1$]

$\Rightarrow 20k + 11\,(3 - 2k) = 0$

$\Rightarrow k = \frac{33}{2}$

Sol.43 (d)

$\left|\frac{z-i}{z+i}\right| =$ purely real

$\arg\left|\frac{z-i}{z+i}\right| = 0$

$\int_2^4 [0]\, dx = 0$

Sol.44 (a)

$S_1 : \sin^{-1} x - \frac{\pi}{2} + \sin^{-1}(-x) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(x) + \sin^{-1}(-x) = \pi$

$0 = \pi$ which is not possible $\therefore$ no solution

$S_2 : \sin^{-1}(x^2 + 4x + 3) + \cos^{-1}(x^2 + 6x + 8) = \frac{\pi}{2}$

$= \sin^{-1}(x^2 + 4x + 3) + \cos^{-1}(x^2 + 4x + 3)$

$\Rightarrow x^2 + 6x + 8 = x^2 + 4x + 3$

$\Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$

$\because x^2 + 4x + 3 = (x+2)^2 - 1 \in [-1, 1]$

at $x = -\frac{5}{2}$

& $x^2 + 6x + 8 = (x+3)^2 - 1 \in [-1, 1]$

at $x = -\frac{5}{2}$

$\therefore x = -\frac{5}{2}$

$S_3 \; \because -1 \le \cos(\sin^{-1} x) \le 1$

and $-1 \le \sin(\cos^{-1} x) \le 1$

$\sin^{-1}\{\cos(\sin^{-1} x)\} + \cos^{-1}\{\sin(\cos^{-1} x)\} = \frac{\pi}{2}$

$S_4 : 2\left[\tan^{-1}\frac{1+2}{1-2} + \pi + \tan^{-1} 3\right]$

$= 2[\pi - \tan^{-1} 3 + \tan^{-1} 3] = 2\pi$

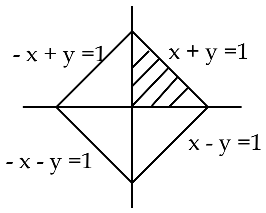
Sol.45 (a)

$S_1 : a_{ji} = j^2 - i^2 = -(i^2 - j^2)$

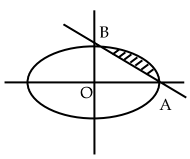
$a_{ji} = -a_{ij}$

Skew-symmetric matrix

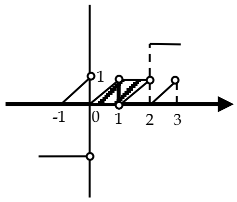
$S_2 : Area = 4\left(\frac{1}{2}.1.1\right) = 2$



$S_3 : Area = \frac{1}{4}(Ellipse\ area) - \Delta OAB = \frac{\pi ab}{4} - \frac{ab}{2}$



$S_4 : Area = 2.\left(\frac{1}{2}1.1\right) = 1$



Sol.46 (a)

$S_1 : [\sin^{-1} x] = \{1 + x^2\} = \{x^2\}$

$$[\sin^{-1} x] = \begin{cases} -2 & , -1 \le x < -\sin 1 \\ -1 & , -\sin 1 \le x < 0 \\ 0 & , 0 \le x < \sin 1 \\ 1 & , \sin 1 \le x < 1 \end{cases}$$

$\therefore$ the only solution is *x = 0*

S_2 : Range of $f(x)$ is whole of R S_2 is true

S_3 : No basic inverse trigonometric function is periodic.

$S_4 : (x^2 - 3x - 10)\ell n^2 (x-3) \ge 0$

i.e. $x^2 - 3x - 10 \ge 0, x = 4$

i.e. $x \in [5, \infty) \cup \{4\}$

$\therefore$ S_4 is false

Sol.47 (a)

$S_1 : x^2 + 4y^2 - 2x - 16y + 13 = 0$

i.e. $(x^2 - 2x + 1) + 4(y^2 - 4y + 4) = 4$

i.e. $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{1} = 1$

$\therefore$ length of latus rectum $= \frac{2 \times 1}{2} = 1$

$\therefore$ S_1 is true

S_2 : For the ellipse $a = 2,\ b = 1$

$\therefore$ $e = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$

$\therefore 2ae = 2.2.\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$\therefore$ S_2 is false

S_3 : Sum of the focal distance $= 2a = 4$

$\therefore S_3$ is true.

S_4 : tangents at the vertices are

$x - 1 = \pm 2$

$x = 3, -1$

$\therefore$ the line $y = 3$ intersect there as

$P(3,3)$ *and* $Q(-1,3)$

A focus is $x = 1+\sqrt{3}, y = 2$

i.e. focus is $S\left(\sqrt{3}+1,\ 2\right)$

slope of PS *is* $\dfrac{1}{2-\sqrt{3}}$, *slope of* QS *is* $\dfrac{1}{-2-\sqrt{3}}$

$\therefore$ Product of slopes $= \dfrac{1}{2-\sqrt{3}} \times \dfrac{1}{-2-\sqrt{3}} = -1$

$\therefore S_4$ is true

Sol.48 (d)

$$f(x) = \frac{x^2+5x-14}{x^2-7x+10} = \frac{x+7}{x-5}$$

$$\therefore\ f(2) = \frac{-9}{3} = -3$$

Sol.49 (b)

$$p = \lim_{n\to\infty}\left[\frac{\prod_{r=1}^{n}\left(n^3+r^3\right)}{n^{3n}}\right]^{1/n}$$

$$\ell n\ p = \lim_{n\to\infty}.\frac{1}{n}\sum_{r=1}^{n}\ell n\left(1+\left(\frac{r}{n}\right)^3\right)$$

$$= \int_0^1 \ell n\left(1+x^3\right)dx = \ell n\ 2 - 3 + 3\lambda$$

Sol.50 (b)

Given,

$$1 \ge \left|z-(4-3i)\right| \ge \begin{cases}|z|-|4-3i| \\ |4-3i|-|z|\end{cases}$$

$\Rightarrow |z| \le 6$ *and* $|z| \ge 4$ $\qquad \Rightarrow 4 \le |z| \le 6 \Rightarrow \alpha = 4,\ \beta = 6$

Let $y = \dfrac{x^4+x^2+4}{x} = x^3 + x + \dfrac{4}{x} \qquad = x^3 + x + \dfrac{1}{x} + \dfrac{1}{x} + \dfrac{1}{x} + \dfrac{1}{x}$

Since $x \in (0,\infty)$, therefore

$x^3, x, \dfrac{1}{x}$ are positive.

Sum will be least when $x^2 = x = \dfrac{1}{x} \quad \Rightarrow x = 1$

$\therefore\quad k = 6$

Hence, $k = \beta$

Sol.51 (c)

Sol.52 (d)

Sol.51 & 52

$f(xy)=xf(y)+yf(x)\forall x,y\in R$ replace x by 1, y by x

$f(x)=f(x)+xf(1)\Rightarrow f(1)=0$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f\left(x\left(1+\frac{h}{x}\right)\right)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{xf\left(1+\frac{h}{x}\right)+\left(1+\frac{h}{x}\right)f(x)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f\left(1+\frac{h}{x}\right)-f(1)}{\frac{h}{x}}+\frac{f(x)}{x}$$

$$f'(x)=f'(1)+\frac{f(x)}{x}$$

$$f'(x)-\frac{f(x)}{x}=f'(1)=1$$

$$\frac{f'(x)}{x}-\frac{f(x)}{x^2}=\frac{1}{x}$$

$$\int d\left(\frac{f(x)}{x}\right)=\int\frac{1}{x}dx$$

$$\frac{f(x)}{x}=\ell nx+c$$

$$f(x)=x\ell nx+cx$$

$$f(1)=0=0+c\Rightarrow c=0$$

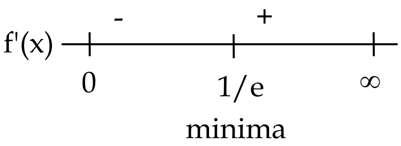
$$f(x)=x\ell nx$$

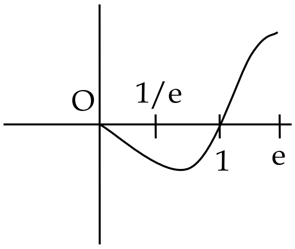
$$\lim_{x\to 0^+}(1+f(x))^{1/x}=\lim_{x\to 0^+}e^{\frac{f(x)}{x}}$$

$$e^{\lim_{x\to 0^+}\ell nx}=e^{-\infty}=0$$

$$f(x)=x\ell nx$$

$$f'(x)=\ell nx+x.\frac{1}{x}=1+\ell nx$$





Sol.53 (b)
Sol.54 (a)

Sol. 53 & 44

Since no point of the parabola is below x-axis

$\therefore a^2 - 4 \le 0$

$\therefore$ maximum value of *a* is 2

Equation of the parabola, when a = 2 is

$y = x^2 + 2x + 1$

It intersects y-axis at (0, 1)

Equation of the tangent at (0, 1) is $y = 2x + 1$

Since $y = 2x + 1$ touches the circle $x^2 + y^2 = r^2$

$\therefore r = \frac{1}{\sqrt{5}}$

Equation of the tangent at (0,1) to the parabola

$y = x^2 + ax + 1$ is $\frac{y+1}{2} = \frac{a}{2}(x+0) + 1$

i.e. $ax - y + 1 = 0$

$\therefore r = \frac{1}{\sqrt{a^2+1}}$

Radius maximum when a = 0

$\therefore$ equation of the tangent is $y = 1$

$\therefore$ slope of the tangent is 0

Sol.55 (a)

Sol.56 (c)

Sol.55 & 56

$$\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1}$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x+1} = x + 1 - \frac{3}{x+1}$$

$$\Rightarrow d\left(\frac{y}{x+1}\right) = \int 1 - \frac{3}{(x+1)^2}\,dx$$

$$\frac{y}{x+1} = x + \frac{3}{x+1} + c$$

$$y = (x+c)(x+1) + 3$$

(2,0) lies on it

$$0 = (2+c)(2+1) + 3$$

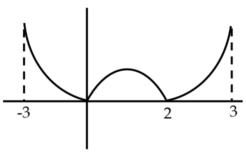
$$2 + c + 1 = 0 \qquad \Rightarrow c = -3$$

$$y = (x-3)(x+1) + 3$$

$$y = x^2 + x - 3x \qquad \Rightarrow y = x^2 - 2x$$

Parabola

$$y = \left|x^2 - 2x\right|$$



Area $= \int_{-3}^{0} (x^2 - 2x)dx + \int_{0}^{2} (2x - x^2)dx$

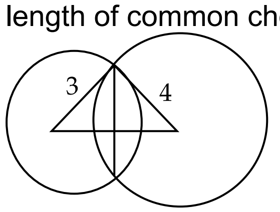
$$+ \int_{2}^{3} (x^2 - 2x)dx$$

$$= \left(\frac{x^3}{3} - x^2\right)\Bigg|_{-3}^{0} + \left(x^2 - \frac{x^3}{3}\right)\Bigg|_{0}^{2} + \left(\frac{x^3}{3} - x^2\right)\Bigg|_{2}^{3}$$

$$= \frac{62}{3}$$

Sol.57 (c)

(P) Let length of common chord be 2a, then



$$\sqrt{9 - a^2} + \sqrt{16 - a^2} = 5$$

$$\sqrt{16 - a^2} = 5 - \sqrt{9 - a^2}$$

$$16 - a^2 = 25 + 9 - a^2 - 10\sqrt{9 - a^2}$$

$$10\sqrt{9 - a^2} = 18 \quad \Rightarrow 100(9 - a^2) = 324$$

i.e. $100a^2 = 576$

$\therefore\ a = \sqrt{\frac{576}{100}} = \frac{24}{10}$

$\therefore 2a = \frac{24}{5} = \frac{k}{5} \Rightarrow\ k = 24$

(Q) Equation of common chord is

$6x + 4y + p + q = 0$

Common chord pass through centre (-2,-6) of circle

$x^2 + y^2 + 4x + 12y + p = 0$

$\therefore p + q = 36$

(R) Equation of the circle is $2x^2 + 2y^2 - 2\sqrt{2}\,x - y = 0$

Let $(\alpha, 0)$ be mid point of a chord. Then equation of the chord is

$2\alpha x - \sqrt{2}(x+\alpha) - \frac{1}{2}(y+0) = 2\alpha^2 - 2\sqrt{2}\ \alpha$

Since it passes through the point $\left(\sqrt{2}, \frac{1}{2}\right)$

$\therefore 2\sqrt{2}\ \alpha - \sqrt{2}\left(\sqrt{2}+\alpha\right) - \frac{1}{4} = 2\alpha^2 - 2\sqrt{2}\ \alpha$

i.e. $8\alpha^2 - 12\sqrt{2}\ \alpha + 9 = 0$

i.e. $\left(2\sqrt{2}\ \alpha - 3\right)^2 = 0$

i.e. $\alpha = \frac{3}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}$ $\therefore$ number of chords is 1

(S) Mid point of AB = (1, 4)

$\therefore$ Equation of perpendicular bisector of AB is x = 1

A diameter is $4y = x + 7$

$\therefore$ Centre of the circle is (1, 2)

$\therefore$ sides of the rectangle are 8 and 4

$\therefore$ area = 32

Sol.58 (a)

(P) $f(x) = |\sin x| + |\cos x|$

Period is $\frac{\pi}{2}$.

(Q) $\vec{P} = \left(\left(\vec{a}\times\vec{b}\right).\vec{c}\right)\vec{b} - \left(\left(\vec{a}\times\vec{b}\right).\vec{b}\right)\vec{c} = \left[\vec{a}\vec{b}\vec{c}\right]\vec{b}$

Similarly $\vec{q} = \left[\vec{a}\vec{b}\vec{c}\right]\vec{c}$ *and* $\vec{r} = \left[\vec{a}\vec{b}\vec{c}\right]\vec{a}$

$\Rightarrow [\vec{p}\vec{q}\vec{r}] = \left[\vec{a}\vec{b}\vec{c}\right]^4 \Rightarrow n = 4$

(R) as $\sqrt{\frac{a^2+2b^2}{a}} > b$, points must lie on major axis

Now $a=\sqrt{\dfrac{a^2+2b^2}{2}} \Rightarrow \dfrac{b^2}{a^2}=\dfrac{1}{2}$

$\therefore\ e=\dfrac{1}{\sqrt{2}} \qquad \Rightarrow k=2$

(S) For ΔABC

$\cot A\ \cot B+\cot B\ \cot C+\cot C\ \cot A=1$

Also $2\cot B=\cot A+\cot C$

$A.M.\geq, G.M$

$\dfrac{\cot A+\cot C}{2}\geq\sqrt{\cot A\cot C}$

$\Rightarrow \cot^2 B\geq \cot A\cot C$

$\geq 1-\cot B(\cot A+\cot C)$

$\geq 1-2\cot^2 B$

$\Rightarrow \cot^2 B\geq\dfrac{1}{3}$

$\Rightarrow (\cot B)_{\min}=\dfrac{1}{\sqrt{3}}$

$\therefore k=3$

Sol.59 (d)

(P) $S=\lim\limits_{n\to\infty}\sum\limits_{n=1}^{n}\dfrac{{}^nC_2}{2^n}$

$$S=\frac{1}{4}+\frac{3}{8}+\frac{6}{16}+\frac{10}{32}+........ \qquad .....(1)$$

$$\frac{S}{2}=\ +\frac{1}{8}+\frac{3}{16}+\frac{6}{32}+......... \qquad ....(2)$$

$$\frac{S}{2}=\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+......... \qquad ....(3)$$

[from (1) - (2)]

$$\frac{S}{4}=\ +\frac{1}{8}+\frac{2}{16}+\frac{3}{32}+.......... \qquad ....(4)$$

$$\frac{S}{4}=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+......... \qquad [From((3)-(4)]$$

$$\Rightarrow \frac{S}{4}=\frac{(1/4)}{1-(1/2)}=\frac{1}{4}.\frac{2}{1}=\frac{1}{2}\Rightarrow S=2$$

(Q) Uncommon roots when $g(x)\neq 0\ \ and\ \ f(x)$ is zero are 2, 9 which are the solutions.

(R) $y=\dfrac{1-2\sin^2 x\cos^2 x}{\sin x\cos x} \quad =\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x}-2\sin x\cos x\ =(\tan x+\cot x)-\sin 2x$

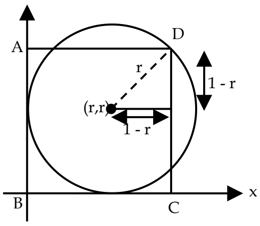
Now, $(\tan x+\cot x)$ is minimum at $x=\pi/4$ and sin 2x is maximum at $x=\pi/4$

$\therefore\ y_{\min}\ \ occurs\ at\ \ x=\pi/4\ \ and\ \ y_{\min}=1$

(S) $r^2 = 2(1-r)^2$

$r^2 - 4r + 2 = 0$

$r = 2 - \sqrt{2}$



Sol.60 (b)

(P) Put *x = 2* in both equation we get

$4 + 2a + b = 0$ ……(i)

$4 + 2c + d = 0$ ……(ii)

(i) – (ii)

$\Rightarrow \dfrac{(b-d)}{(c-a)} = 2$

(Q) No. of arrangements $= \dfrac{6!}{2!} = 3.5!$

(R) Last non zero digit in 21! Can be obtained by using exponent method

$No.\ of\ 2's = \left[\dfrac{21}{2}\right] + \left[\dfrac{21}{4}\right] + \left[\dfrac{21}{8}\right] + \left[\dfrac{21}{16}\right] = 18$

No. of $5's = \left[\dfrac{21}{5}\right] = 4$

$\therefore$ No. of zeroes = 4

No. of $3's = \left[\dfrac{21}{3}\right] + \left[\dfrac{21}{9}\right] = 9$

$\therefore$ Last non zero digit will be obtained

from $2^{14} \times 3^9 \times 7^3 \times 11 \times 13 \times 15 \times 17 \times 19 = 4$

(S) $\therefore$ $37^{n+2} + 16^{n+1} + 30^n$

$= (35+2)^{n+2} + (14+2)^{n+1} + (28+2)^n$

$= 7k + 2^{n+2} + 2^{n+1} + 2^n$

$= 7k + 2^n.7$

$\therefore$ remainder = 0